CHAPTER 1. MATERIALS ENGINEERING CONCEPTS

1.2. Strength at rupture = **450 MPa**
Toughness = $(450 \times 0.003)/2$ = **0.675 MPa**

1.3. $A = 201.06 \text{ mm}^2$
$\sigma = 0.945$ GPa
$\varepsilon_A = 0.002698$ m/m
$\varepsilon_L = -0.000625$ m/m
E = 350.3 GPa
ν = 0.23

1.4. $A = \pi d^2/4 = 17671 \text{ mm}^2$
$\sigma = P / A = -665000/17671 = -37.6$ MPa
$E = \sigma / \varepsilon = 55$ GPa
$\varepsilon_A = \sigma / E = -37.6$ MPa/55 GPa $= -0.000684$ m/m
$\Delta L = \varepsilon_A L_o = -0.000683$ m/m(0.300 m) $= -0.2$ mm
$L_f = \Delta L + L_o = 300$ mm $- 0.2$ mm = **299.8 mm**
$\nu = -\varepsilon_L / \varepsilon_A = 0.35$
$\varepsilon_L = \Delta d / d_o = -\nu \varepsilon_A = -0.35(-0.000684$ m/m$) = 0.000239$ m/m
$\Delta d = \varepsilon_L d_o = 0.000239 \times 0.150 = 0.036$ mm
$d_f = \Delta d + d_o = 150$ mm + 0.036 mm = **150.036 mm**

1.5. $A = \pi d^2/4 = 78.54 \text{ mm}^2$
$\sigma = P / A = (9 \text{ kN})/(78.54 \text{ mm}^2) = 115$ MPa (Less than the yield strength. Within the elastic region)
$E = \sigma / \varepsilon = 70$ GPa
$\varepsilon_A = \sigma / E = 115$ MPa/70 GPa $= 0.001643$ m/m
$\Delta L = \varepsilon_A L_o = 0.001643$ m/m $(0.300) = 0.5$ mm
$L_f = \Delta L + L_o = 300$ mm + 0.5 mm = **300.5 mm**
$\nu = -\varepsilon_L / \varepsilon_A = 0.33$
$\varepsilon_L = \Delta d / d_o = -\nu \varepsilon_A = -0.33\ (0.001643$ m/m$) = 0.000542$
$\Delta d = \varepsilon_L d_o = -0.000542(10$ mm$) = -0.005$ mm
$d_f = \Delta d + d_o = 10$ mm $- 0.005$ mm = **9.995 mm**

1.6. L_x =30 mm, L_y = 60 mm, L_z = 90 mm

$\sigma_x = \sigma_y = \sigma_z = \sigma$ = 100 MPa

E = 70 GPa

ν = 0.333

$\varepsilon_x = [\sigma_x - \nu (\sigma_y + \sigma_z)] / E$

$\varepsilon_x = [100 \times 10^6 - 0.333 (100 \times 10^6 + 100 \times 10^6)] / 70 \times 10^9 = 4.77 \times 10^{-4} = \varepsilon_y = \varepsilon_z = \varepsilon$

$\Delta L_x = \varepsilon \times L_x = 4.77 \times 10^{-4} \times 30 = 0.01431$ mm

$\Delta L_y = \varepsilon \times L_y = 4.77 \times 10^{-4} \times 60 = 0.02862$ mm

$\Delta L_z = \varepsilon \times L_z = 4.77 \times 10^{-4} \times 90 =$ **0.04293 mm**

ΔV = New volume − Original volume = $[(L_x - \Delta L_x)(L_y - \Delta L_y)(L_z - \Delta L_z)] - L_x L_y L_z$

= (30 − 0.01431) (60 − 0.02862) (90 − 0.04293)] − (30 × 60 × 90) = 161768 − 162000

= −**232 mm³**

1.7. L_x = 100 mm, L_y = 100 mm, L_z = 100 mm

$\sigma_x = \sigma_y = \sigma_z = \sigma$ = 100 MPa

E = 7 GPa

ν = 0.39

$\varepsilon_x = [\sigma_x - \nu (\sigma_y + \sigma_z)] / E$

$\varepsilon_x = [100 - 0.39(100 + 100)]/7000 = 0.003 = \varepsilon_y = \varepsilon_z = \varepsilon$

$\Delta L_x = \varepsilon \text{ x } L_x = 0.003 \times 100 = 0.3$ mm

$\Delta L_y = \varepsilon \text{ x } L_y = 0.003 \times 100 = 0.3$ mm

$\Delta L_z = \varepsilon \text{ x } L_z = 0.003 \times 100 = 0.3$ mm

ΔV = New volume − Original volume = $[(L_x - \Delta L_x)(L_y - \Delta L_y)(L_z - \Delta L_z)] - L_x L_y L_z$

= [(100 − 0.4)(100 − 0.4)(100 − 0.4)] − (100 × 100 × 100)

= −**8973.03 mm³**

1.8. $\varepsilon = 0.3 \times 10^{-16} \sigma^3$

At σ = 345 MPa, $\varepsilon = 0.3 \times 10^{-16} (345000)^3 = 1.232$ m/m.

Secant modulus = $\frac{\Delta\sigma}{\Delta\varepsilon} = \frac{345,000}{1.232}$ = **2.80×10^5 kPa**

$\frac{d\varepsilon}{d\sigma} = 0.9 \times 10^{-16} \sigma^2$

At σ = 345,000 kPa, $\frac{d\varepsilon}{d\sigma} = 0.9 \text{ x } 10^{-16} (345,000)^2 = 1.071 \times 10^{-5}\ \text{kPa}^{-1}$

Tangent modulus = $\frac{d\sigma}{d\varepsilon} = \frac{1}{1.071 \times 10^{-5}}$ = **9.34×10^4 kPa**

1.9. ε_{axial} = 0.05 / 50 = 0.001 m/m

$\varepsilon_{lateral} = -\nu \times \varepsilon_{axial}$ = –0.33 × 0.001 = 0.00033 m/m

$\Delta d = \varepsilon_{lateral} \times d_0$ = –0.00825 mm (Contraction)

1.10. L = 380 mm
D = 10 mm
P = 24.5 kN
$\sigma = P/A = P/\pi r^2$
σ = 24,500 N/ π (5 mm)^2= 312,000 N/mm^2 = 312 Mpa
The copper and aluminum can be eliminated because they have stresses larger than their yield strengths as shown in the table below.
For steel and brass, $\delta = \frac{PL}{AE} = \frac{24{,}500 lbx380mm}{\pi(5mm)^2 E(kPa)} = \frac{118{,}539}{E(MPa)}$ mm

Material	Elastic Modulus (MPa)	Yield Strength (MPa)	Tensile Strength (MPa)	Stress (MPa)	δ (mm)
Copper	110,000	248	289	312	
Al. alloy	70,000	255	420	312	
Steel	207,000	448	551	312	0.573
Brass alloy	101,000	345	420	312	1.174

The problem requires the following two conditions:
a) No plastic deformation ⇒ Stress < Yield Strength
b) Increase in length, δ < 0.9 mm
The only material that satisfies both conditions is **steel**.

1.11. $\sigma = \frac{F}{A_0} = \frac{31{,}000\ N}{\pi\left(\frac{15.24\times10^{-3}m}{2}\right)^2} = 169.9 = 170\ MPa$

This stress is less than the yield strengths of all metals listed.

$\Delta l = \frac{\sigma L_0}{E}$

Material	E (GPa)	Yield Strength (MPa)	Tensile Strength (MPa)	ΔL (mm)
Steel alloy 1	180	860	502	0.378
Steel alloy 2	200	400	250	0.340
Titanium alloy	110	900	730	0.618
Copper	117	220	70	0.581

Only the steel alloy 1 and steel alloy 2 have elongation less than 0.45 mm.

1.12. a. $E = \sigma / \varepsilon$ = 40,000 / 0.004 = **10 x 10^6 psi**

b. Tangent modulus at a stress of 310 MPa is the slope of the tangent at that stress = **32405 MPa**

c. Yield stress using an offset of 0.002 strain = **338 MPa**

d. Maximum working stress = Failure stress / Factor of safety = 338 MPa/1.5 = **225 MPa**

1.13.a. Modulus of elasticity within the linear portion = **116667 MPa**

b. Yield stress at an offset strain of 0.002 m/m ≈ **480 MPa**

c. Yield stress at an extension strain of 0.005 m/m ≈ **475 MPa**

d. Secant modulus at a stress of 525 MPa ≈ **124106 MPa**

e. Tangent modulus at a stress of 525 MPa ≈ **41368 MPa**

1.14. a. Modulus of resilience = the area under the elastic portion of the stress strain curve = **0.4309 MPa**

b. Toughness = the area under the stress strain curve (using the trapezoidal integration technique) ≈ **4.76 MPa**

c. σ = 275 MPa, this stress is within the elastic range, therefore, E = **116667 MPa**

ε_{axial} = 275/116667 = 0.002 m/m

$$\nu = -\frac{\varepsilon_{lateral}}{\varepsilon_{axial}} = -\frac{-0.00057}{0.002} = \mathbf{0.285}$$

d. The permanent strain at 480 MPa = **0.0018 m/m**

1.15.

	Material A	Material B
a. Proportional limit	**320 MPa**	**280 MPa**
b. Yield stress at an offset strain of 0.002 m/m	**434 MPa**	**358 MPa**
c. Ultimate strength	**820 MPa**	**500 MPa**
d. Modulus of resilience	**0.448 MPa**	**0.48 MPa**
e. Toughness	**56.5 MPa**	**52 MPa**
f.	Material B is more ductile as it undergoes more deformation before failure	

1.16. Assume that the stress is within the linear elastic range.

$$\sigma = \varepsilon.E = \frac{\delta.E}{l} = \frac{7.6x105{,}000}{250} = 3{,}192 \text{ MPa}$$

Thus $\sigma > \sigma_{yield}$

Therefore, the applied stress is not within the linear elastic region and it is not possible to compute the magnitude of the load that is necessary to produce the change in length based on the given information.

1.17. At σ = 410 MPa, ε = σ / E = 400/200000 = 0.002 m/m

a. For a strain of 0.001 m/m:

σ = ε E = 0.001 × 200000 = 200 MPa **(**for both i and ii)

b. For a strain of 0.004 m/m:

σ = 410 MPa (for i)

σ = 410 + 15000(0.004 – 0.002) = 440 MPa **(**for ii)

1.18. a. Slope of the elastic portion = 600/0.003 = 2x10^5 MPa

Slope of the plastic portion = (800-600)/(0.07-0.003) = 2,985 MPa

Strain at 650 MPa = 0.003 + (650-600)/2,985 = 0.0198 m/m

Permanent strain at 650 MPa = 0.0198 – 650/(2x10^5) = **0.0165 m/m**

b. Percent increase in yield strength = 100(650-600)/600 = **8.3%**

c. The strain at 625 MPa = 625/(2x10^5) = **0.003125 m/m**

This strain is elastic.

1.19. a. $\sigma_{max} = \frac{F}{A_o} = \frac{39{,}872\ N}{100\ x\ 10^6\ m^2} =$ 0.000399 Pa = 398 MPa

b. $E = \frac{\sigma}{\varepsilon} = \frac{\sigma\ x\ L_o}{\Delta L} = \frac{\sigma\ x\ L_o}{(L - L_o)}$

$E\ x\ (L - L_o) = \sigma\ x\ L_o$

$110x10^3\ MPa\ x\ (67.21\ mm - L_o) = 398\ MPa\ x\ L_o$

$L_o = 66.97\ mm$

1.20. $\boldsymbol{\varepsilon_a = \frac{-\varepsilon_l}{\nu} = \frac{\frac{-\Delta d}{d}}{\nu} = \frac{-\Delta d}{d\,\nu}}$

$$\boldsymbol{E = \frac{\sigma_a}{\varepsilon_a} = \frac{\frac{F}{(\frac{\pi d^2}{4})}}{\frac{-\Delta d}{d\,\nu}} = \frac{-4Fd\,\nu}{\pi d^2 \Delta d}}$$

$$F = -\frac{-d\Delta d\pi E}{4\nu}$$

$$\mathrm{F} = \frac{-(19 \times 10^{-3}\ \mathrm{m})(-3.0 \times 10^{-6}\ \mathrm{m})(\pi)\left(110 \times 10^{9}\,\frac{\mathrm{N}}{\mathrm{m}^2}\right)}{4(0.35)} = 14{,}070\ N$$

1.21. See Sections 1.2.3, 1.2.4 and 1.2.5.

1.22. The stresses and strains can be calculated as follows:

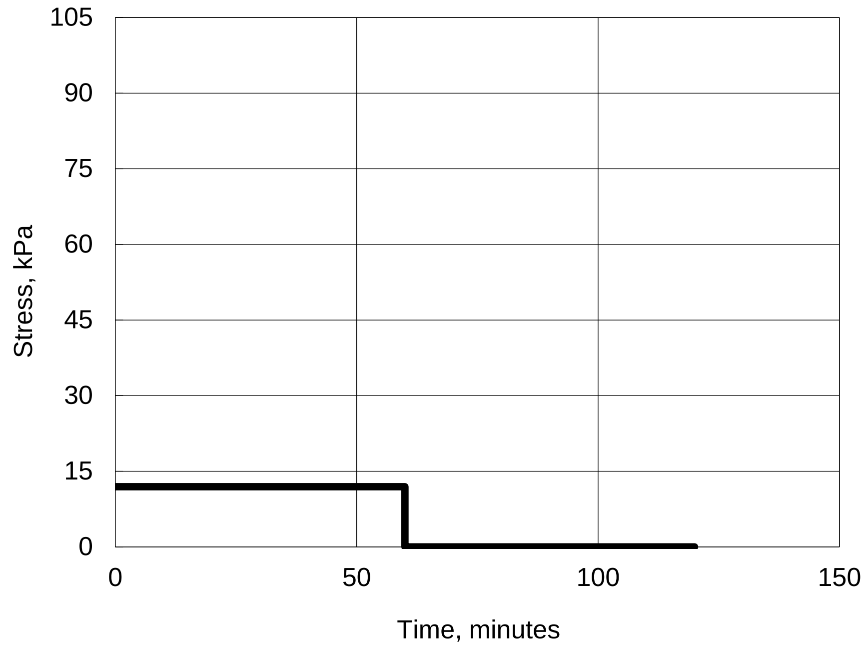
$\sigma = P/A_o = 700/(\pi \times 50^2) = 0.089$ MPa

$\varepsilon = (H_o - H)/H_o = (150 - H)/150$

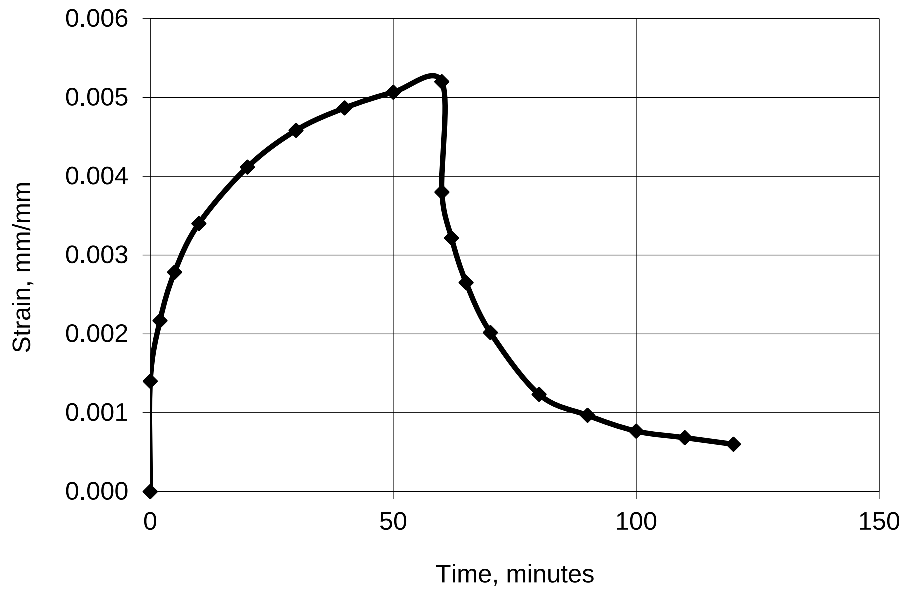
The stresses and strains are shown in the following table:

Time (min.)	H (mm)	Strain (m/m)	Stress (kPa)
0	150	0.00000	89.17197
0.01	149.79	0.00140	89.17197
2	149.67	0.00217	89.17197
5	149.58	0.00278	89.17197
10	149.49	0.00340	89.17197
20	149.38	0.00412	89.17197
30	149.31	0.00458	89.17197
40	149.27	0.00487	89.17197
50	149.24	0.00507	89.17197
60	149.22	0.00520	89.17197
60.01	149.43	0.00380	0
62	149.52	0.00322	0
65	149.60	0.00265	0
70	149.70	0.00202	0
80	149.82	0.00123	0
90	149.85	0.00097	0
100	149.88	0.00077	0
110	149.90	0.00068	0
120	149.91	0.00060	0

a. Stress versus time plot for the asphalt concrete sample



Strain versus time plot for the asphalt concrete sample



b. Elastic strain = **0.0014 mm/mm**

c. The permanent strain at the end of the experiment = **0.0006 mm/mm**

d. The phenomenon of the change of specimen height during static loading is called **creep** while the phenomenon of the change of specimen height during unloading called is called **recovery.**

1.23. See Figure 1.12(a).

1.24. a. For $F \leq F_o$: $\delta = F.t / \beta$
For $F > F_o$, movement

b. For $F \leq F_o$: $\delta = F / M$
For $F > F_o$: $\delta = F / M + (F - F_o)\, t / \beta$

1.25 See Section 1.2.7.

1.26. a. For P = 5 kN
Stress = P / A = 5000 / (π x 5^2) = 63.7 N/mm^2 = 63.7 MPa
Stress / Strength = 63.7 / 290 = 0.22
From Figure 1.16, an **unlimited number** of repetitions can be applied without fatigue failure.

b. For P = 11 kN
Stress = P / A = 11000 / (π x 5^2) = 140.1 N/mm^2 = 140.1 MPa
Stress / Strength = 140.1 / 290 = 0.48
From Figure 1.16, N ≈**700**

1.27 See Section 1.2.8.

1.28.

Material	Specific Gravity
Steel	7.9
Aluminum	2.7
Aggregates	2.6 - 2.7
Concrete	2.4
Asphalt cement	1 - 1.1

1.29. See Section 1.3.2.

1.30. $\delta L = \alpha_L \; x \; \delta T \; x \; L$ = 12.5E-06 *x* (115-15) *x* 200/1000 = 0.00025 m = 250 microns
Rod length = L + δL = 200,000+ 250 = **200,250 microns**

Compute change in diameter linear method

$\delta d = \alpha_d \ x \ \delta T \ x \ d$ = 12.5E-06 x (115-15) x 20 = 0.025 mm

Final d = **20.025 mm**

Compute change in diameter volume method

$\delta V = \alpha_V \ x \ \delta T \ x \ V$ = (3 x 12.5E-06) x (115-15) x π $(10/1000)^2$ x 200/1000 = 2.3562 x 10^{11} m^3

Rod final volume = $V + \delta V = \pi r^2 L + \delta V$= 6.28319 x 10^{13} + 2.3562 x 10^{11} = 6.31 x 10^{13} m^3

Final d = **20.025 mm**

There is no stress acting on the rod because the rod is free to move.

1.31. Since the rod is snugly fitted against two immovable nonconducting walls, the length of the rod will not change, **L = 200 mm**

From problem 1.25, δL = 0.00025 m

$\varepsilon = \delta L$ / L = 0.00025 / 0.2 = 0.00125 m/m

$\sigma = \varepsilon$ E = 0.00125 x 207,000 = **258.75 MPa**

The stress induced in the bar will be compression.

1.32. a. The change in length can be calculated using Equation 1.9 as follows:

$\delta L = \alpha_L \ x \ \delta T \ x \ L$ = 1.1E-5 x (5 - 40) x 4 = **-0.00154 m**

b. The tension load needed to return the length to the original value of 4 meters can be calculated as follows:

$\varepsilon = \delta L$ / L = -0.00154/ 4 = -0.000358 m/m

$\sigma = \varepsilon$ E = -0.000358 x 200,000 = -77 MPa

P = σ x A = -77 x (100 x 50) = -385,000 N = **-385 kN (tension)**

c. Longitudinal strain under this load = **0.000358 m/m**

1.33. If the bar was fixed at one end and free at the other end, the bar would have contracted and no stresses would have developed. In that case, the change in length can be calculated using Equation 1.9 as follows.

$\delta L = \alpha_L \times \delta T \times L = 0.000009 \times (-20 - 40) \times 1250 = -0.000675$ m

$\varepsilon = \delta L$ / L = 0.000675/1.25 = 0.0005 m/m

Since the bar is fixed at both ends, the length of the bar will not change. Therefore, a tensile stress will develop in the bar as follows.

$\sigma = \varepsilon E = -0.0005 \times 35{,}000 = -17.5$ MPa

Thus, the tensile strength should be larger than **17.5 MPa** in order to prevent cracking.

1.34. See Section 1.7.

1.35. See Section 1.7.1

1.36. H_o: $\mu \geq 32.4$ MPa

H_1: $\mu < 32.4$ MPa

$\alpha = 0.05$

$$T_o = \frac{\bar{x} - \mu}{(\sigma / \sqrt{n})} = -3$$

Degree of freedom = $\nu = n - 1 = 15$

From the statistical t-distribution table, $T_{\alpha, \upsilon} = T_{0.05, 15} = -1.753$

$T_o < T_{\alpha, \upsilon}$

Therefore, **reject** the hypothesis. The contractor's claim is not valid.

1.37.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{20} x_i}{20} = \frac{785.67}{20} = 39.28\text{ MPa}$$

$$s = \left(\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} \right)^{1/2} = \left(\frac{\sum_{i=1}^{20} (x_i - 39.28)^2}{20-1} \right)^{1/2} = 3.94\text{ MPa}$$

$$\text{Coefficient of Variation} = 100\left(\frac{s}{\bar{x}}\right) = 100\left(\frac{3.94\text{ MPa}}{39.28}\right) = 10.03\%$$